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NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

For Supervisor's use only

Level 3 Statistics and Modelling, 2009

90644 Solve equations

Credits: Four
9.30 am Friday 20 November 2009

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables Booklet L3–STATF.

You should answer ALL the questions in this booklet.

Show ALL working for ALL questions.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only	Achievement Criteria				
Achievement		**Achievement with Merit**		**Achievement with Excellence**	
Solve equations.	☐	Solve problems involving equations.	☐	Analyse or interpret the outcome or the process used to solve equations or linear programming problems.	☐
Overall Level of Performance	☐				

You are advised to spend 55 minutes answering the questions in this booklet.

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QUESTION ONE

(a) A manufacturer produces 500 g bags containing a mixture of hazelnuts, macadamia nuts and scorched almonds.

In each bag, the weight of the scorched almonds is twice the weight of the hazelnuts.

The combined weight of the hazelnuts and the macadamia nuts is 100 g less than the weight of the scorched almonds.

The following system of equations represents the situation above:

$$x + y + z = 500$$
$$y = 2x$$
$$x + z = y - 100$$

Solve the system of equations to determine the weight of macadamia nuts in each 500 g bag.

(b) The manufacturer imports the three varieties of nuts by the kilogram.

The most recent order totalled 80 kg.

The weight of scorched almonds was three times the combined weight of the macadamia nuts and hazelnuts.

The weight of scorched almonds was four kilograms less than eight times the weight of the hazelnuts.

Set up and solve a system of equations to find how many kilograms of hazelnuts were in the order.

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(c) Two mathematics students, Kyle and Rebecca, were attempting to solve the following system of equations:

$$\begin{aligned} 2x + 4y - 3z &= 9 \\ -18x - 11y + 2z &= -31 \\ 4x + 3y - z &= 8 \end{aligned}$$

Kyle finds that (4,–5,–7) is a solution.
Rebecca finds that (–5,13,11) is a solution.

Explain how the equations are related, and give a geometric interpretation of this situation.
You do not need to attempt to solve the equations to answer this question.

QUESTION TWO

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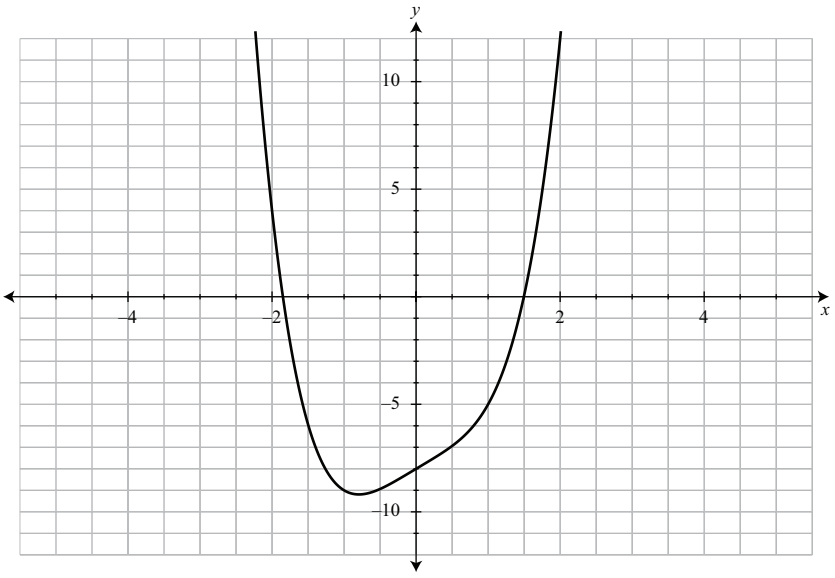
The graph of the function $f(x) = x^4 + 2x - 8$ is drawn below.



(a) The equation $x^4 + 2x - 8 = 0$ has a root between $x_0 = -2$ and $x_1 = -1$.

Using the two x values above as the starting values, complete TWO iterations of the **bisection method** to find an approximation to this root.

Show each iterate.

(b) The equation $x^4 + 2x - 8 = 0$ also has a positive root.

Using $x = 1$ as the starting value, use the **Newton-Raphson method** to find the positive root of this function correct to 1 decimal place.

Show each iterate.

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(c) The graph of a function $y = g(x)$ is drawn below.

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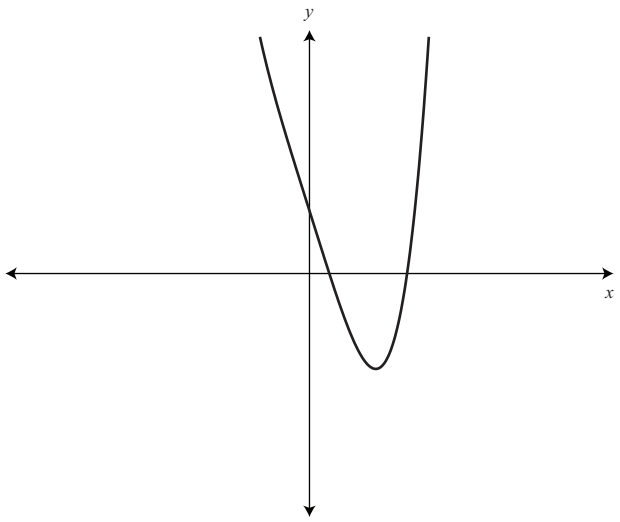


Explain the circumstances in which using the Newton-Raphson method would **not** find the **greater** root of $g(x) = 0$.

QUESTION THREE

Nick is a farmer. He is contracted to deliver a minimum of 10 tonnes and a maximum of 20 tonnes of corn to a local factory. Nick also is contracted to deliver at least 15 tonnes, but no more than 30 tonnes of tomatoes to the same factory.

For every tonne of corn Nick produces, he needs 8 hectares of land and, for every tonne of tomatoes he produces, he needs 11 hectares. He has only 440 hectares available for the two crops.

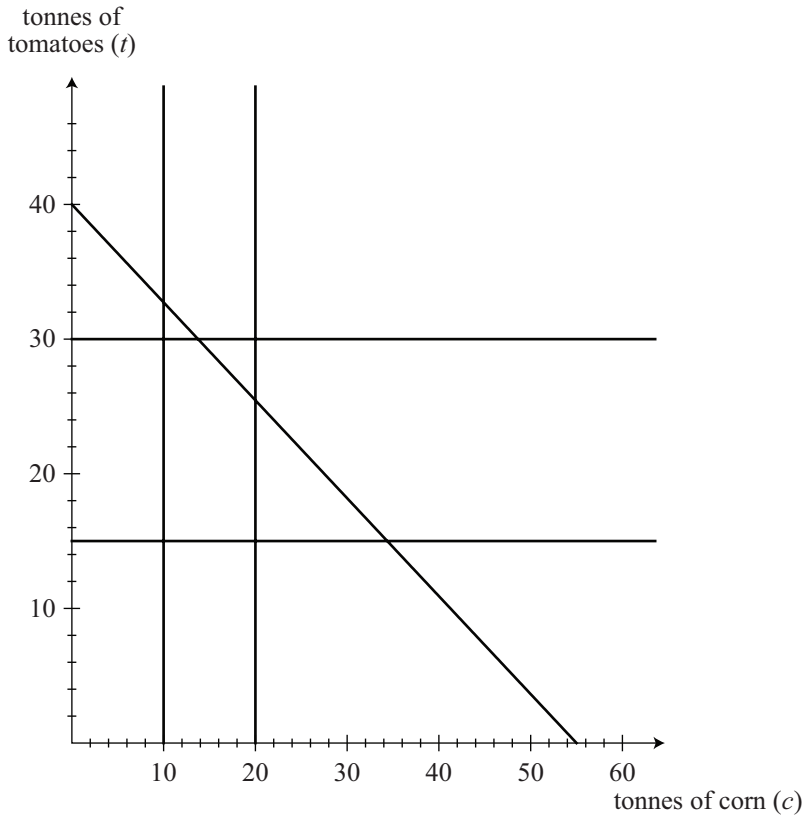
Nick's income from each tonne of corn is $1 250 and from each tonne of tomatoes is $3 000.

Let c = the number of tonnes of corn produced
and t = the number of tonnes of tomatoes produced.

A linear programming problem for Nick's situation has the following constraints:

$10 \le c \le 20$ $\quad$ $15 \le t \le 30$ $\quad$ $8c + 11t \le 440$

The graph below is drawn to assist you with this question.



Nick's income ($) is given by the objective function:

$Income = 1250c + 3000t$

(a) Use the information on the opposite page to calculate how many tonnes of corn and tomatoes Nick should plant to maximise his income.

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Question Three continues on the following page.

(b) The factory changes its contracts, and now requires producers to grade their product(s) before sending them to the factory. In return, the factory will increase the income for each tonne of corn to $2000 and the income for each tonne of tomatoes to $3500.

Each tonne of corn that Nick produces will take 9 hours of labour to grade the produce, and each tonne of tomatoes will take 8 hours of labour. Nick only has 360 hours of labour available for grading these two crops.

How many tonnes of corn and tomatoes should Nick now produce to maximise his income? Justify your answer.

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(.) A linear programming problem may have multiple solutions.

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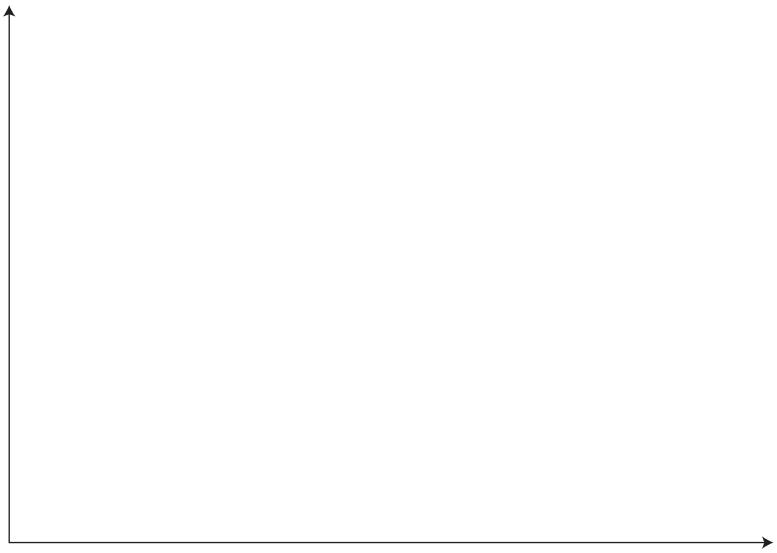
On the axes below, show how this occurs and explain how your diagram shows this.

If you need to redraw this graph, use the grid at the bottom of page 12.



Extra paper for continuation of answers if required.
Clearly number the question.

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Question number	

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If you need to redraw the graph from page 11, draw it on the grid below. Make sure it is clear which graph you want marked.