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NZQA
NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

For Supervisor's use only

Level 3 Statistics and Modelling, 2008

90644 Solve equations

Credits: Four
9.30 am Monday 24 November 2008

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables Booklet L3–STATF.

You should answer ALL the questions in this booklet.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–15 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only	**Achievement Criteria**		
Achievement	**Achievement with Merit**	**Achievement with Excellence**	
Solve equations.	Solve problems involving equations.	Analyse or interpret the outcome or the process used to solve equations or linear programming problems.	
Overall Level of Performance			

You are advised to spend 65 minutes answering the questions in this booklet.

Assessor's use only

QUESTION ONE

A landscape gardener is building a new garden. The gardener has ordered 20 m^3 of stones in three different sizes, at a total cost of $950. The stones are supplied in three sizes: large, medium and small. The price of each size is given in the table below.

	Large	**Medium**	**Small**
Price (per m^3)	$40	$50	$55

The following system of equations represents information about the stones ordered:

$$40x + 50y + 55z = 950$$
$$x + y + z = 20$$
$$y = 2x$$

Solve the system of equations to find the volume of small stones that was ordered.

QUESTION TWO

Assessor's use only

The gardener wants to grow three types of plants (A, B, and C) in the new garden.

The total cost of purchasing 40 of plant A, 60 of plant B, and 100 of plant C is $3 600.
The price of each plant A is three dollars more than the price of each plant C.
The price of each plant B is six dollars more than twice the price of each plant C.

Set up and solve a system of equations to find the cost of each plant C.

QUESTION THREE

Assessor's use only

The gardener plans to plant another garden using three other types of plants (D, E, and F).

Plant D costs $15 per plant, plant E costs $5 per plant, and plant F costs $20 per plant.
Seventy-five plants are to be bought, at a total cost of $500.
The number of plant E to be purchased is the same as the combined total of the number of plant D and twice the number of plant F.

Set up a system of equations for the above information and solve them, clearly justifying your answer and carefully explaining what your result means about the gardener's plans.

QUESTION FOUR

Assessor's use only

The equation $x^3 - 5x - 4 = 0$ has a root that lies between $x = 2$ and $x = 3$.

Complete two iterations of EITHER the Newton-Raphson method OR the bisection method to find an approximation to this root.

Use $x_o = 2$ as the starting value if you use the Newton-Raphson method.

Use $x_o = 2$ and $x_1 = 3$ as the starting values if you use the bisection method.

Show each iterate.

QUESTION FIVE

Assessor's use only

Use EITHER the Newton-Raphson method OR the bisection method to find the **smallest root** of the equation $x^5 - 2x + 1 = 0$.

You must state your starting value(s), show the results of each iteration and give your answer correct to one decimal place.

If you want to draw a graph, use the grid on page 12.

QUESTION SIX

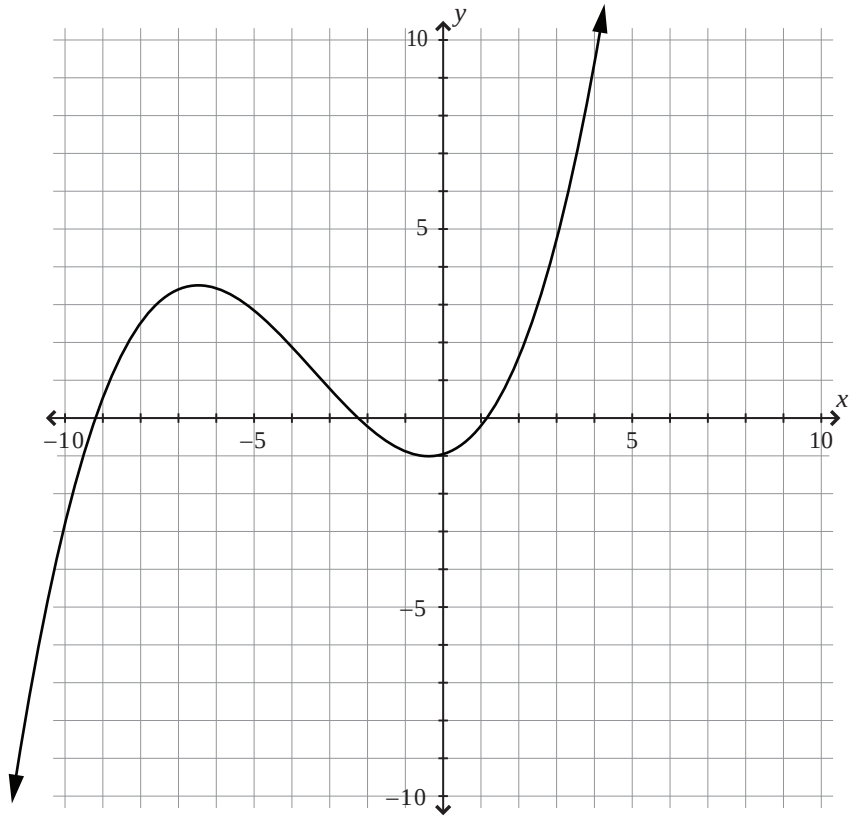
Assessor's use only

The diagram below shows the graph of $y = f(x)$.

A student wanted to use the Newton-Raphson method to find the roots of the equation $f(x) = 0$.

The student decided to take $x_o = -6$ as the starting value.

Show which root the Newton-Raphson method will converge to with a starting value of $x_o = -6$. Clearly explain your reasoning.



If you want to redraw your answer, use the grid on page 12.

QUESTION SEVEN

Assessor's use only

A grower produces tomatoes for a local factory. He has heard that by growing artichokes and selling them at the Farmer's Market he can make more money. Artichokes are labour-intensive to grow and he needs to decide whether they are worth the investment.

Growing tomatoes requires 10 hours of labour per hectare.
Growing artichokes requires 20 hours of labour per hectare.
The grower has 1 200 hours of labour available for these two crops.

The grower has 90 hectares altogether to plant.
If he wants to keep his contract with the factory he must plant at least 30 hectares of tomatoes. The grower feels that planting anything less than 10 hectares of artichokes would not be giving the new crop a fair trial.

A linear programming problem for the grower's situation has the following constraints.

A: $x \geq 30$
B: $y \geq 10$
C: $x + y \leq 90$
D: $10x + 20y \leq 1\,200$

(a) Show these constraints on the axes below, and show the feasible region for the problem. Three of the lines that you need have been drawn for you.

If you need to redraw this graph, use the grid on page 13.

(b) The grower receives \$10 000 per hectare for his tomatoes and expects to receive \$25 000 per hectare for his artichokes, so his income \$$I$ is given by the equation $I = 10\,000x + 25\,000y$.

Find how many hectares of each crop the grower should plant to maximise his income.

Assessor's use only

QUESTION EIGHT

Assessor's use only

A farmer has a small farm of 120 hectares. Currently she has sheep on the property, but she has decided to convert some of the land to farm dairy cows.

She knows that each sheep needs 0.3 hectares of pasture, and each cow needs 0.4 hectares.

The farmer's fertiliser costs are $180 per year for each sheep, and she has been told that each cow will cost $150 per year in fertiliser. She does not want to spend more than $54 000 on fertiliser per year.

The farmer does not want to have more than 250 cows on the farm.

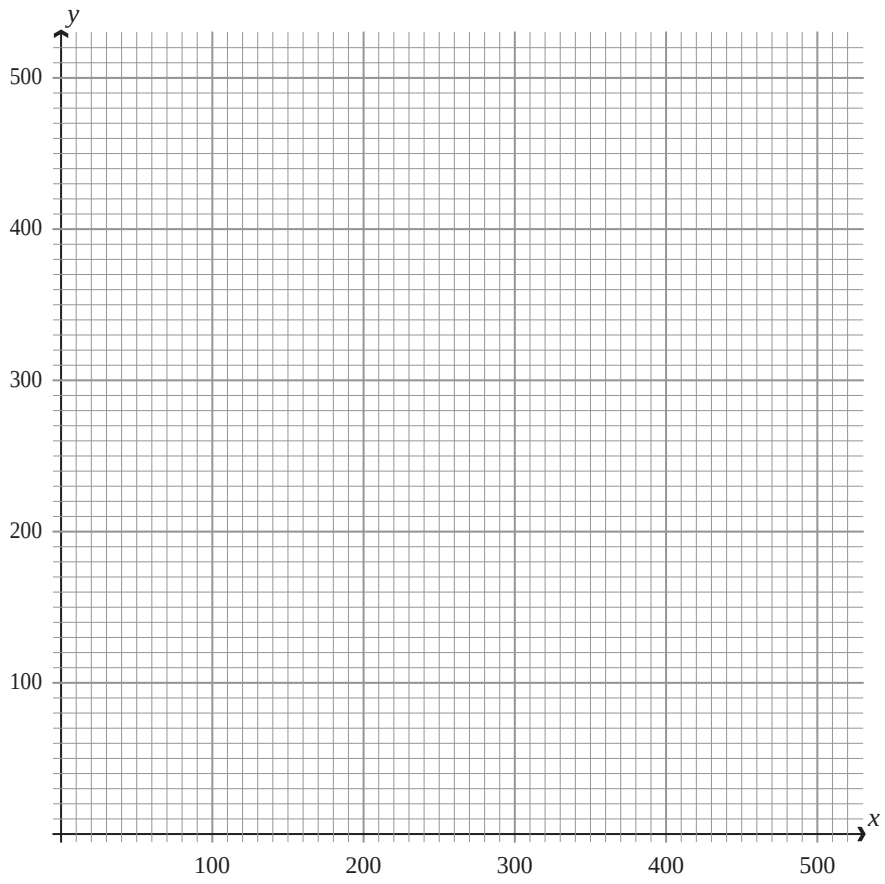
Current income is $700 per year for each sheep, and expected income for each cow is $850 per year.

You may use the axes on page 11 to help you answer the following questions.

(a) How many sheep and how many cows should the farmer have on the property to maximise her total income? Justify your answer.

(b) Calculate the new maximum income the farmer could obtain if there was a 20% reduction in her total income from sheep. Justify your answer.

Assessor's use only

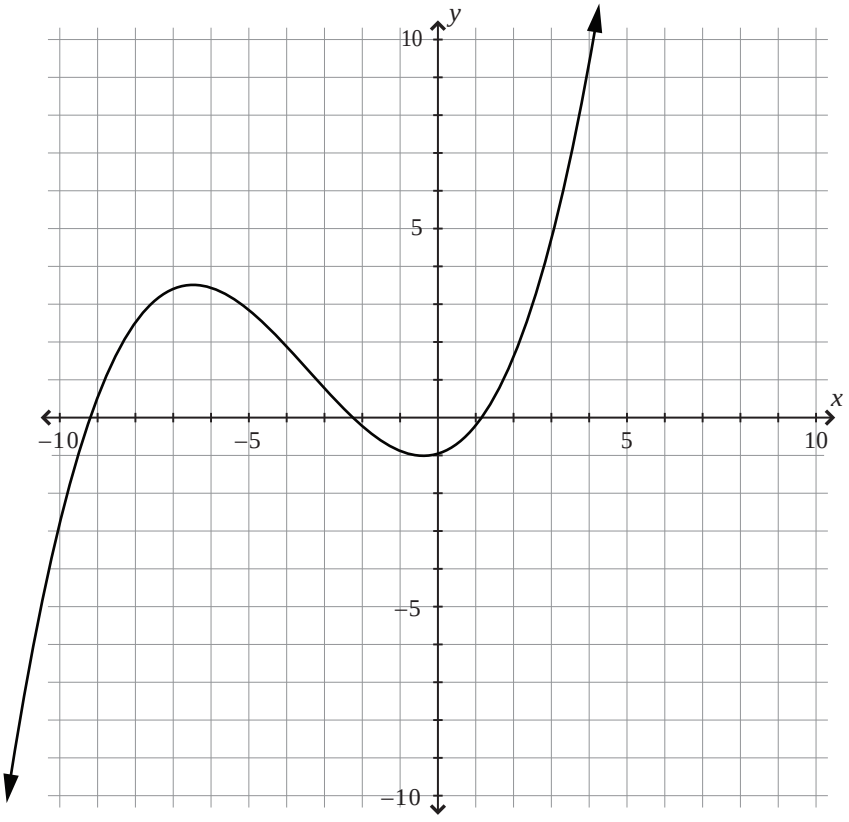


If you need to redraw this graph, use the grid on page 13.

Use the grid below if you need to draw a graph as part of your answer to Question Five.

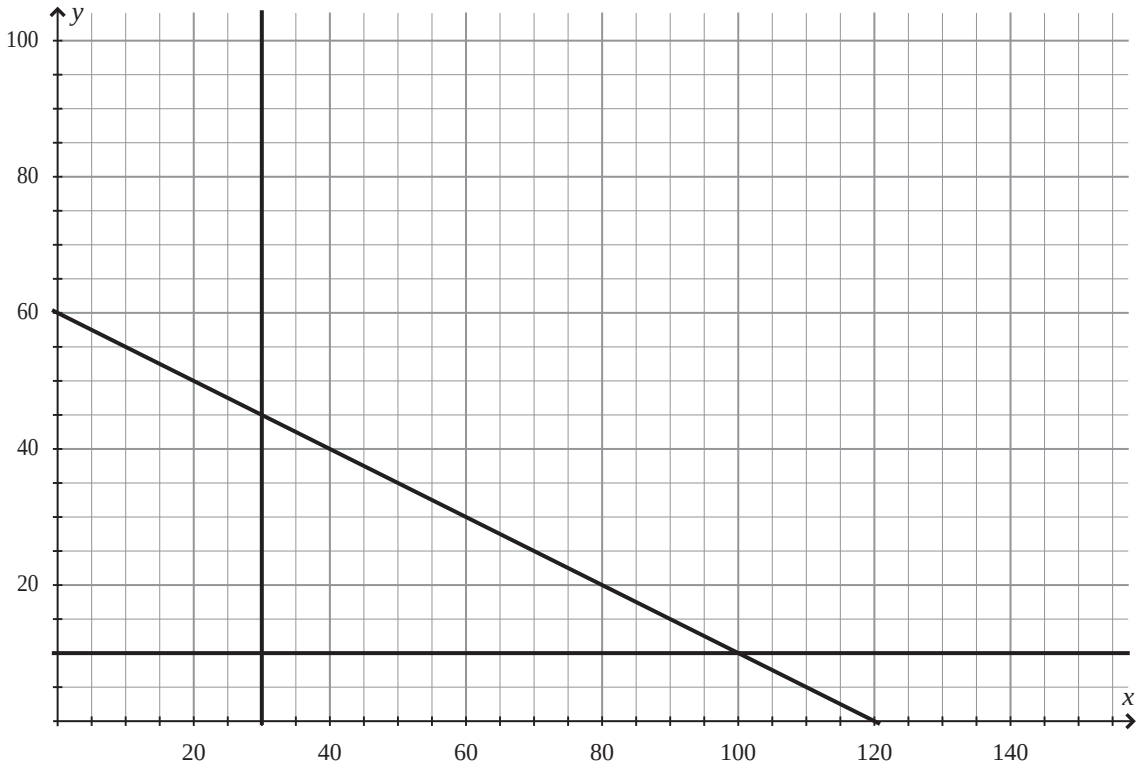
Assessor's use only

Use the grid below if you need to redraw your answer to Question Six.

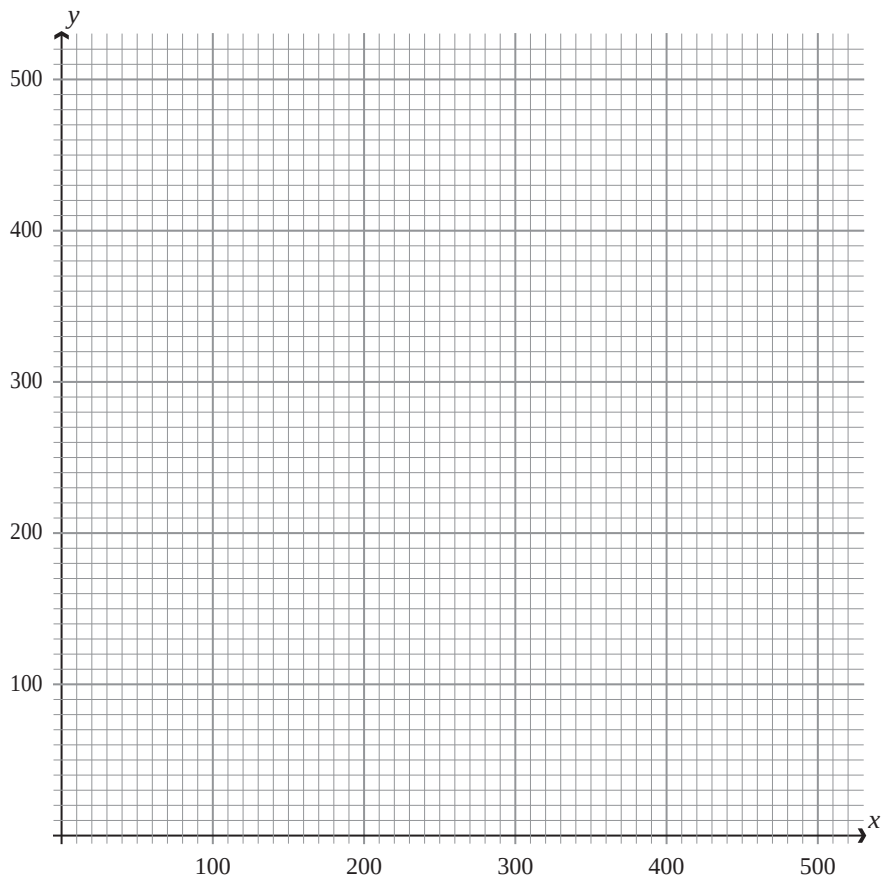


Use the grid below if you need to redraw your graph for Question Seven.

Assessor's use only



Use the grid below if you need to redraw your graph for Question Eight.



Extra paper for continuation of answers if required.
Clearly number the question.

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